

# Instability analysis of Marangoni convection for absorption process accompanied by heat transfer

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## Abstract

The objective of this paper is to analyze the onset of Marangoni convection for absorption process accompanied by heat transfer using propagation theory. Ammonia–water is used as a representative solution pair for the absorption process and 2-ethyl-1-hexanol is considered an additive. The re-modified solutal Marangoni number  $MBs$  is linearly related to the modified solutal Biot number  $Bs^*$ . Rearranging this relation, a useful correlation of the solutal Marangoni number  $Ms$  with respect to the solutal Biot number  $Bs$  is obtained in a linear form. It is found that there is a critical  $Bs$  causing the most unstable state of fluid layer. The critical time  $\tau_c$  to mark the onset of Marangoni convection can be predicted using the correlation developed in the present study. The results from the present study will provide a guideline to enhance heat and mass transfer performance during the absorption process.

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**Keywords:** Marangoni convection; Absorption; Heat transfer; Propagation theory; Additive

## 1. Introduction

When an initially quiescent fluid experiences absorption or desorption of a solute through a free surface, the surface tension of the fluid varies with the concentration and temperature of the fluid at the surface. This variation of the surface tension can cause Marangoni convection. After Pearson's [1] theoretical analysis of the thermal Marangoni convection, many studies on Marangoni instability have been performed theoretically or experimentally in the heat transfer systems with different boundary conditions [2–4]. Under the linear stability theory, Sternling and Scriven [5] analyzed solutal Marangoni instabilities of two immiscible fluid layers with mass transfer through the interface. Their onset criteria of Marangoni convection for linear basic concentration profiles have been applied to other sys-

tems [6,7]. Using propagation theory for evaporation system, Kang et al. [8] analyzed Brian's [7] system and showed that theoretical results obtained by their numerical simulation agree well with experimental ones obtained by Brian et al. [9].

When certain surfactants are added to the liquid in small quantities, the heat and mass transfer rate can be significantly enhanced beyond the diffusion and conduction controlled rates of the stable system. Many studies on these phenomena have been performed in LiBr/H<sub>2</sub>O absorption process [10–12]. Kang et al. [13] conducted experiments on the visualization of Marangoni convection in NH<sub>3</sub>/H<sub>2</sub>O absorption system with several additives and reported the meaningful results of the effect of additives on the initiation of solutal Marangoni convection.

The objective of this study is to clarify the onset criteria of Marangoni convection for absorption process accompanied by heat transfer based on the propagation theory. The results from the present study will provide a guideline to enhance heat and mass transfer performance during the absorption process.

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### Nomenclature

$a$	dimensionless wave number
$A, B$	coefficients in Eq. (48)
$B_s$	solubility Biot number
$B_t$	thermal Biot number
$C$	concentration
$C_{eq}$	equilibrium concentration
$d$	liquid depth
$D$	differential operator
$D_l$	solubility diffusivity
$h$	transfer coefficient in gas phase
$i$	imaginary number
$Le$	Lewis number
$MB_s$	re-modified solubility Marangoni number
$MB_t$	re-modified thermal Marangoni number
$Ms$	solubility Marangoni number
$Mt$	thermal Marangoni number
$P$	pressure
$S$	surface tension
$Sc$	Schmidt number
$t$	time
$T$	temperature
$\vec{U}$	velocity vector
$\vec{u}$	dimensionless velocity vector
$W$	vertical velocity
$w$	dimensionless vertical velocity

$x, y, z$  coordinates

### Greek symbols

$\alpha$	thermal diffusivity
$\beta$	basic concentration gradient
$\Delta\phi$	solubility penetration depth
$\phi$	dimensionless concentration
$\gamma$	surface tension gradient
$\mu$	viscosity
$\nu$	kinematic viscosity
$\theta$	dimensionless temperature
$\rho$	density
$\tau$	dimensionless time
$\zeta$	similarity variable

### Subscripts

0	basic state
1	perturbed state
c	critical condition
i	initial state
s	solubility quantity
t	thermal quantity

### Superscript

*	amplitude function
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## 2. Marangoni convection

In general, the solute in solution lowers the surface tension at the liquid–vapor interface. However in solution involving certain additives, as the concentration of the solute increases, the surface tension of the liquid also increases. When this happens, Marangoni convection driven by surface tension variation occurs resulting in significant enhancement of the heat and mass transfer rate. Using the linear stability theory, Sternling and Scriven [5] and Brian [6] reported that both the sign of the surface tension gradient with respect to the solute concentration and the direction of the mass transfer mainly affect the onset of Marangoni convection.

In this present study, the absorbate transfers from the gas phase to the liquid phase as shown in Fig. 1. Applying the results of the above works to the absorption system considered here, the basic condition of the onset of Marangoni convection is expressed as follow:

$$\beta\gamma_s < 0, \quad (1)$$

where  $\beta(= \partial C / \partial Z)$  and  $\gamma_s(= \partial S / \partial C)$  denote the basic concentration gradient and the surface tension gradient with respect to the solute concentration.

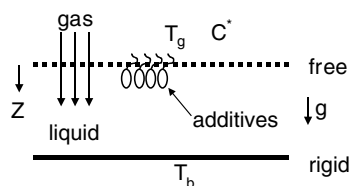


Fig. 1. Schematic diagram of system considered here.

## 3. Formulation of problem

A horizontal liquid layer considered here is bounded an upper free surface at  $Z = 0$  and a lower rigid plane at  $Z = d$ , as shown in Fig. 1. At time  $t = 0$  the absorbate at different temperature from liquid layer, i.e., ammonia in the gas phase begins to be absorbed into the absorbant with additives, i.e., water involving 2-ethyl-1-hexanol. The bulk liquid is incompressible and Boussinesque solution. Both the Soret and Rayleigh effects caused by absorption process are neglected. No absorption heat is generated at the surface.

Under the above approximations, the governing equations to represent Marangoni convection are given by

$$\nabla \cdot \vec{U} = 0, \tag{2}$$

$$\rho \left( \frac{\partial}{\partial t} + \vec{U} \cdot \nabla \right) \vec{U} = -\nabla P + \mu \nabla^2 \vec{U}, \tag{3}$$

$$\left( \frac{\partial}{\partial t} + \vec{U} \cdot \nabla \right) C = D_1 \nabla^2 C, \tag{4}$$

$$\left( \frac{\partial}{\partial t} + \vec{U} \cdot \nabla \right) T = \alpha \nabla^2 T, \tag{5}$$

$$S = S_i + \gamma_s(C - C_i) + \gamma_t(T - T_i), \tag{6}$$

where  $\vec{U}$ ,  $\rho$ ,  $P$ ,  $\mu$ ,  $C$ ,  $D_1$ ,  $T$ ,  $\alpha$ ,  $S$  and  $\gamma_i (= \partial S / \partial T)$  denote the velocity vector, the liquid density, the pressure, the liquid viscosity, the solute concentration, the solutal diffusivity in the liquid, the temperature, the thermal diffusivity of the liquid, the surface tension and the surface tension gradient with respect to the temperature, respectively. The subscript  $i$ ,  $s$  and  $t$  represent the quantity of the initial state, the solutal quantity and the thermal one.

In the present system, the basic profiles of concentration and temperature of the solution can be represented as the following dimensionless equation

$$\frac{\partial \phi_0}{\partial \tau} = \frac{\partial^2 \phi_0}{\partial z^2}, \quad \frac{\partial \theta_0}{\partial \tau} = Le \frac{\partial^2 \theta_0}{\partial z^2}, \tag{7-1, 2}$$

with the following proper initial and boundary conditions,

$$\phi_0 = 0, \quad \theta_0 = 0 \quad \text{at } \tau = 0, \tag{8}$$

$$-\frac{\partial \phi_0}{\partial z} = Bs(1 - \phi_0), \quad -\frac{\partial \theta_0}{\partial z} = Bt(1 - \theta_0) \tag{9}$$

at  $z = 0$ ,

$$\frac{\partial \phi_0}{\partial z} = 0, \quad \theta_0 = 0 \quad \text{at } z = 1, \tag{10}$$

where  $z$ ,  $\tau$ ,  $\phi_0$  and  $\theta_0$  denote the dimensionless vertical distance, the dimensionless time, the dimensionless basic concentration of the solute, and the dimensionless basic temperature, respectively. The subscript 0 denotes the basic quantity. Length is non-dimensionalized by the liquid depth  $d$ , the time by  $d^2/D_1$ , and the basic concentration by  $C_{eq}$  and the basic temperature by  $\Delta T (= T_g - T_b)$ . In the case of no-absorption heat generation at the surface, the boundary conditions of temperature and concentration are given as Eq. (9), which is a general expression at the surface. It can be applied for both cases of equilibrium and non-equilibrium states. The dimensionless parameters  $Le$ ,  $Bs$  and  $Bt$  are defined as

$$\text{Lewis number } Le = \frac{\alpha}{D_1}, \tag{11}$$

$$\text{solutal Biot number } Bs = \frac{h_s d}{D_1}, \tag{12}$$

$$\text{thermal Biot number } Bt = \frac{h_t d}{\alpha}, \tag{13}$$

where  $h_s$  and  $h_t$  denote the mass-transfer coefficient and the heat-transfer one, respectively.

Under the semi-infinite approximation, the similarity solutions for the above equations are well known [14],

$$\begin{aligned} \phi_0 &= \text{erfc} \left( \frac{z}{2\sqrt{\tau}} \right) - \exp(Bs \times z + \tau Bs^2) \\ &\quad \times \text{erfc} \left( \frac{z}{2\sqrt{\tau}} + Bs\tau^{1/2} \right) \\ &= \text{erfc} \left( \frac{\zeta}{2} \right) - \exp(Bs^* \zeta + Bs^{*2}) \text{erfc} \left( \frac{\zeta}{2} + Bs^* \right) \\ &\quad \text{for } \tau < 0.01, \end{aligned} \tag{14}$$

$$\begin{aligned} \theta_0 &= \text{erfc} \left( \frac{z}{2\sqrt{Le\tau}} \right) - \exp(Bt \times z + Le\tau Bt^2) \\ &\quad \times \text{erfc} \left( \frac{z}{2\sqrt{Le\tau}} + Bt\sqrt{Le\tau} \right) \\ &= \text{erfc} \left( \frac{\zeta}{2\sqrt{Le}} \right) - \exp(Bt^* \zeta + LeBt^{*2}) \\ &\quad \times \text{erfc} \left( \frac{\zeta}{2\sqrt{Le}} + \sqrt{Le}Bt^* \right) \quad \text{for } \tau < 0.01, \end{aligned} \tag{15}$$

where similarity variable  $\zeta = z/\tau^{1/2}$ , modified solutal Biot number  $Bs^* = \tau^{1/2}Bs$  and modified thermal Biot number  $Bt^* = \tau^{1/2}Bt$ . Here  $Bs^*$  and  $Bt^*$  are assumed to be constant. The above similarity solutions are almost the same with the exact solutions obtained without any approximation for  $\tau < 0.01$ . By this reason they have been used to study the convective instability analysis of the horizontal fluid for deep-pool systems. Over a certain absorbing rate exceeding the critical value, Marangoni convection will set in. The important dimensionless parameters are identified as

$$\text{solutal Marangoni number } Ms = \frac{\gamma_s d}{\mu D_1} C_{eq}, \tag{16}$$

$$\text{thermal Marangoni number } Mt = \frac{\gamma_t d}{\mu \alpha} \Delta T, \tag{17}$$

$$\text{Schmidt number } Sc = \frac{\nu}{D_1}. \tag{18}$$

The Marangoni numbers  $Ms$  and  $Mt$  are a measure of the ratio of the driving (the imposed concentration difference  $C_{eq}$ ; the temperature one  $\Delta T$ ) and the damping (the viscosity and the solute diffusion; the viscosity and the conduction) in the system and are the most important parameters in Marangoni convection.

#### 4. Linear stability theory

Under the normal mode approximation, the dimensionless disturbance quantities  $\vec{u}_1$ ,  $\phi_1$  and  $\theta_1$  can be expressed as follow:

$$(\vec{u}_1, \phi_1, \theta_1) = [\vec{u}_1^*(\tau, z), \phi_1^*(\tau, z), \theta_1^*(\tau, z)] \exp[i(a_x x + a_y y)], \tag{19}$$

where  $\vec{u}_1, \phi_1$  and  $\theta_1$  are the amplitude functions, and  $i, a_x$  and  $a_y$  denote the imaginary number, the wave number in the  $x$ -direction, and the wave number in the  $y$ -direction.  $x$  and  $y$  denote the dimensionless horizontal coordinates. The perturbed velocity has the scale of  $D_1/d$ . The perturbed concentration has been non-dimensionalized by  $C_{eq}$  and the perturbed temperature by  $\Delta T$  with the same manner of the basic ones. Under the linear stability theory, the dimensionless amplitude equations for the vertical velocity amplitude  $w_1^*$ , concentration  $\phi_1^*$  and temperature  $\theta_1^*$  are obtained respectively as follow

$$\left[ \frac{1}{Sc} \frac{\partial}{\partial \tau} - \left( \frac{\partial^2}{\partial z^2} - a^2 \right) \right] \left( \frac{\partial^2}{\partial z^2} - a^2 \right) w_1^* = 0, \tag{20}$$

$$\frac{\partial \phi_1^*}{\partial \tau} + w_1^* \frac{\partial \phi_0}{\partial z} = \left( \frac{\partial^2}{\partial z^2} - a^2 \right) \phi_1^*, \tag{21}$$

$$\frac{\partial \theta_1^*}{\partial \tau} + w_1^* \frac{\partial \theta_0}{\partial z} = Le \left( \frac{\partial^2}{\partial z^2} - a^2 \right) \theta_1^*, \tag{22}$$

where the dimensionless wave number  $a$  denotes  $(a_x^2 + a_y^2)^{1/2}$ . Neglecting the solute accumulation, the surface diffusion and the absorption heat in the absorbed layer, the proper boundary conditions are given by

$$w_1^* = 0 \quad \text{at } z = 0, \tag{23}$$

$$a^2 (Ms \phi_1^* + LeMt \theta_1^*) = - \frac{\partial^2 w_1^*}{\partial z^2} \quad \text{at } z = 0, \tag{24}$$

$$\frac{\partial \phi_1^*}{\partial z} = Bs \phi_1^* \quad \text{at } z = 0, \tag{25}$$

$$\frac{\partial \theta_1^*}{\partial z} = Bt \theta_1^* \quad \text{at } z = 0, \tag{26}$$

$$w_1^* = \frac{\partial w_1^*}{\partial z} = \frac{\partial \phi_1^*}{\partial z} = \theta_1^* = 0 \quad \text{at } (z = 1). \tag{27}$$

Eq. (24) means that the tangential shear stress is balanced by surface tension driven by the solute concentration change and the temperature one at the upper free surface.

**5. Scaling analysis**

Foster [15] showed that for deep-pool system the onset time to mark convective motion is related to the penetration depth other than liquid depth. In the small time region, applying the scaling analysis to Eqs. (21), (22) and (24) the following relations are possible

$$w_1^* \frac{\Delta \phi_0}{A_\phi} \sim \frac{\phi_1^*}{A_\phi^2}, \tag{28}$$

$$w_1^* \frac{\Delta \theta_0}{A_\phi} \sim \frac{\theta_1^*}{A_\phi^2}, \tag{29}$$

$$(Ms \phi_1^* + LeMt \theta_1^*) \sim - \frac{w_1^*}{A_\phi}, \tag{30}$$

where  $A_\phi$  represents the dimensionless concentration penetration depth. The basic concentration and temperature gradients are obtained from Eqs. (14) and (15) as follow

$$\frac{\Delta \phi_0}{A_\phi} \sim Bs \exp(Bs^* + Bs^{*2}) \operatorname{erfc}(0.5 + Bs^*), \tag{31}$$

$$\frac{\Delta \theta_0}{A_\phi} \sim Bt \exp(Bt^* + LeBt^{*2}) \operatorname{erfc}(0.5/\sqrt{Le} + \sqrt{Le}Bt^*). \tag{32}$$

Substituting Eqs. (31) and (32) into Eqs. (28) and (29) and rearranging Eq. (30) the following relation between non-dimensional parameters yield

$$MBs + MBt/Le = Ms^* Bs^* \exp(Bs^* + Bs^{*2}) \operatorname{erfc}(0.5 + Bs^*) + Mt^* Bt^* \exp(Bt^* + LeBt^{*2}) \times \operatorname{erfc}(0.5/\sqrt{Le} + \sqrt{Le}Bt^*), \tag{33}$$

where the modified solutal Marangoni number  $Ms^* = \sqrt{\tau} Ms$  and the modified thermal Marangoni number  $Mt^* = \sqrt{\tau} Mt$ . The re-modified solutal Marangoni number  $MBs$  and the re-modified thermal one  $MBt$  represent the first term and the second term times Lewis number  $Le$  in the right-hand side of the above equation respectively. These re-modified parameters obtained by the scaling analysis can be identified as

re-modified solutal Marangoni number

$$MBs = \frac{\gamma_s d^2}{\mu D_1} \left( \frac{\partial C_0}{\partial Z} \right) \Big|_{Z=A_c}, \tag{34}$$

re-modified thermal Marangoni number

$$MBt = \frac{\gamma_t d^2}{\mu \alpha} \left( \frac{\partial T_0}{\partial Z} \right) \Big|_{Z=A_c}. \tag{35}$$

Considering Eqs. (16), (17), (34) and (35), Marangoni number characterizing the absorption system can be defined with overall concentration difference or the basic concentration gradient for absorption process.

**6. Propagation theory**

For small time region, using similarity variable  $\zeta$ , the coordinate of system can be transformed from  $(\tau, z)$ -frame to  $(\tau, \zeta)$ -frame. In this sense the dimensionless velocity in  $(\tau, \zeta)$ -frame  $w^*$  can be expressed as follow

$$w^*(\zeta) = \frac{W_1^*}{W_{Ac}} = \sqrt{\tau} w_1^*(\tau, z), \tag{36}$$

where  $W_1^*$  is the disturbance velocity and  $W_{\Delta c} (= \Delta c/t)$  is the growth velocity of solutal penetration depth in which  $\Delta c$  and  $t$  denote the solutal penetration depth and time. With this procedure, the following relations are possible:

$$[w_1^*, \phi_1^*, \theta_1^*] = [w^*(\zeta)/\sqrt{\tau}, \phi^*(\zeta), \theta^*(\zeta)]. \tag{37}$$

Inserting the above forms into Eqs. (20)–(27) yield

$$\left[ (D^2 - a^{*2})^2 + \frac{1}{2Sc} (\zeta D^3 - a^{*2} \zeta D + 2a^{*2}) \right] w^* = 0, \tag{38}$$

$$\left( D^2 + \frac{1}{2} \zeta D - a^{*2} \right) \phi^* - w^* D \phi_0 = 0, \tag{39}$$

$$Le \left( D^2 + \frac{1}{2Le} \zeta D - a^{*2} \right) \theta^* - w^* D \theta_0 = 0 \tag{40}$$

with the boundary conditions

$$w^* = 0 \quad \text{at } \zeta = 0, \tag{41}$$

$$D^2 w^* = -a^* (Ms^* \phi^* + Le Mt^* \theta^*) \quad \text{at } \zeta = 0, \tag{42}$$

$$D \phi^* = Bs^* \phi^* \quad \text{at } \zeta = 0, \tag{43}$$

$$D \theta^* = Bt^* \theta^* \quad \text{at } \zeta = 0, \tag{44}$$

$$w^* = Dw^* = D\phi^* = \theta^* = 0 \quad \text{for } \zeta \rightarrow \infty, \tag{45}$$

where  $D(\cdot) = d(\cdot)/d\zeta$ ,  $a^* = a\tau^{1/2}$ . All these dimensionless groups having the superscript \* are assumed to be eigenvalues through  $\tau$ . Then, Eqs. (38)–(45) are functions of the similarity variable  $\zeta$  only. For a given  $Sc$ ,  $Le$ ,  $Bs^*$ ,  $Bt^*$ ,  $Mt^*$  and  $a^*$  the minimum value of  $Ms^*$  is found numerically.

### 7. Results and discussion

With the propagation theory the marginal stability curves for various  $Sc$  are obtained as shown in Fig. 2. It is found that the minimum values of  $Ms^*$  slightly decrease with increasing  $Sc$  for other fixed parameters and the curves become almost the same for  $Sc > 500$ . This means that the increase of  $Sc$  makes the fluid layer be more unstable and the effect of  $Sc$  on the criteria of solutal Marangoni convection instability is not significant for a larger value than 500.

For an absorption system without heat transfer, the re-modified solutal Marangoni number  $MBs$  is linearly related to the modified solutal Biot number  $Bs^*$  as shown in Fig. 3. The following linear correlation is found within the bound of 1% error:

$$MBs = 5.36 + 7.43Bs^*. \tag{46}$$

The above equation for non-linear basic profile system is similar to Nield's result (1964) for linear basic profile system, which is  $Mt = 32.073Bt$  for a large  $Bt$ .

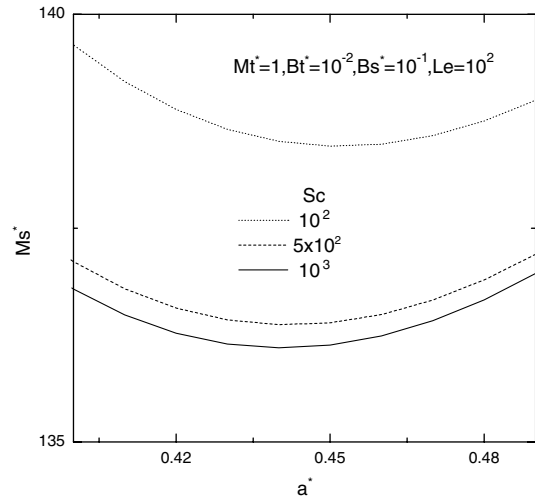


Fig. 2. Marginal stability curves for various  $Sc$ .

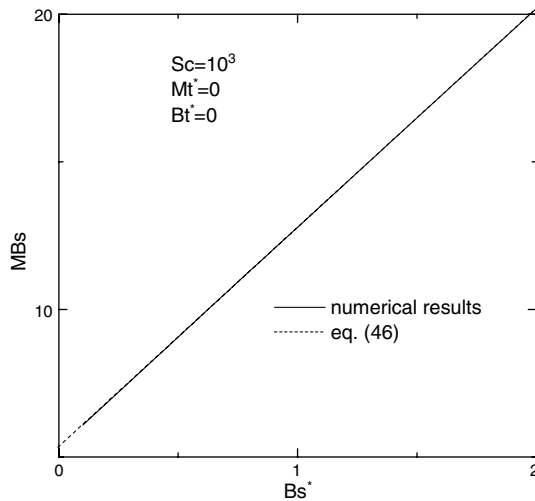


Fig. 3. Re-modified solutal Marangoni number  $MBs$  versus modified solutal Biot number  $Bs^*$  with no heat transfer.

Rearranging Eq. (46) to  $Ms^*$  instead of  $MBs$ , the following equation is obtained:

$$Ms^* = \frac{5.36 + 7.43Bs^*}{Bs^* \exp(Bs^* + Bs^{*2}) \operatorname{erfc}(0.5 + Bs^*)}. \tag{47}$$

It is plotted in the solid line and the values obtained by simulation are featured with the circles in Fig. 4 respectively. Fig. 4 shows that  $Ms^*$  has the minimum at the value of  $Bs^*$  near 0.85. From the definition of  $Ms^*$ , it is obvious that when  $Ms^*$  is minimum,  $\tau_c$  is also minimum with given conditions. It is known that the shorter  $\tau_c$  is, the faster Marangoni convection sets in. Therefore the critical  $Bs^*$  at which the minimum value of  $Ms^*$  is

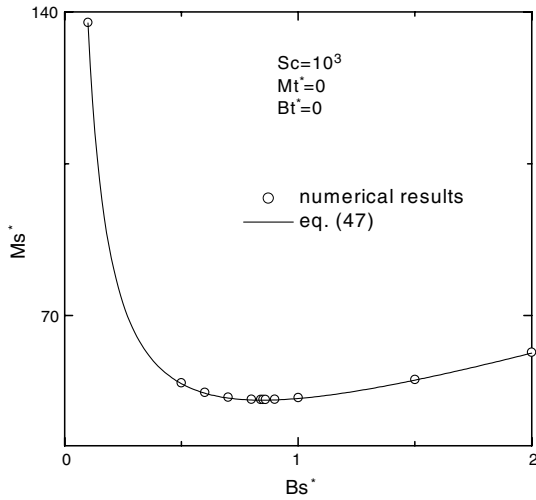


Fig. 4. Modified solutal Marangoni number  $Ms^*$  versus modified solutal Biot number  $Bs^*$  with no heat transfer.

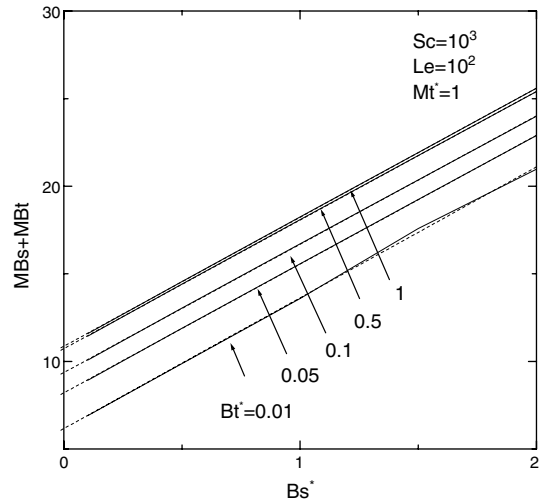


Fig. 6. The sum of re-modified Marangoni numbers versus the solutal Biot number  $Bs^*$  for various  $Bt^*$ .

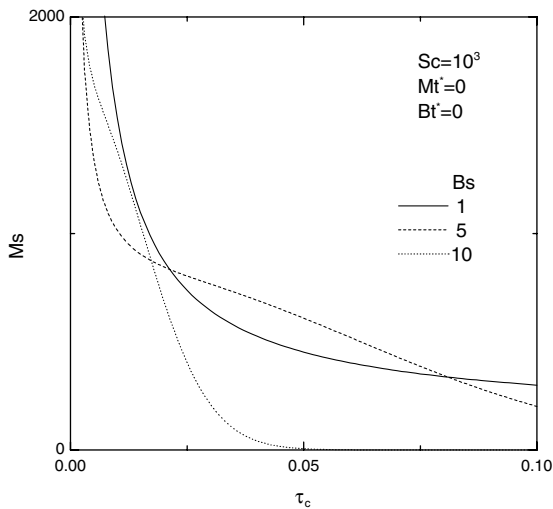


Fig. 5. The effect of  $Ms$  on the critical time  $\tau_c$  for various  $Bs$  with no heat transfer.

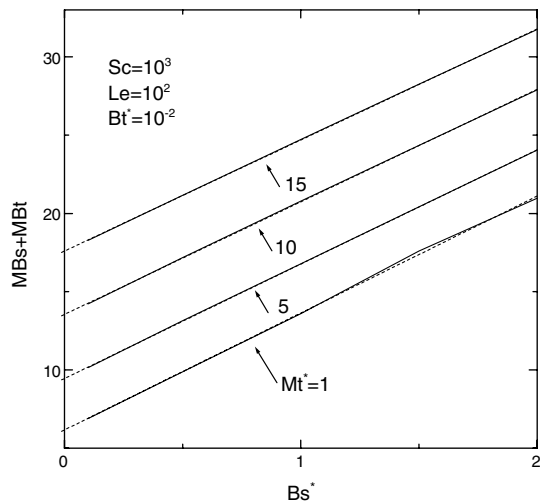


Fig. 7. The sum of re-modified Marangoni numbers versus the solutal Biot number  $Bs^*$  for various  $Mt^*$ .

obtained makes the fluid layer most unstable with a given condition.

Comparing with Pearson's result [1]  $Ms$  becomes infinite instead of a certain value for  $Bs = 0$ . Pearson used Marangoni number based on the basic gradient. On the other hand, Marangoni number used in the present study is based on the overall concentration difference. Regarding the difference of definition, Rabin [16] pointed out that Marangoni number defined by Pearson included the Biot number.

Fig. 5 shows the effect of  $Ms$  on the critical time  $\tau_c$  for various  $Bs$ . It is represented that there is a pair of  $Ms$  and

a critical value of  $Bs$  to make the fluid layer most unstable. For example  $Bs$  of 5 is the critical value for the larger values of  $Ms$  than 900.

The results in the sense of linear relation of the re-modified parameters to  $Bs^*$  are shown in Fig. 6 for various  $Bt^*$  and Fig. 7 for various  $Mt^*$ , respectively. According to the definition of  $MBt$ , it varies with  $Mt^*$  and  $Bt^*$ . In Fig. 6, the effect of  $Bs^*$  on the sum of  $MBs$  and  $MBt$  are obtained with a fixed  $Mt^*$  but a variable  $Bt^*$ . In Fig. 7, these are calculated with a fixed  $Bt^*$  but a variable  $Mt^*$ . Figs. 6 and 7 show that the values of  $MBs + MBt$  for given  $Bt^*$  and  $Mt^*$  with other fixed

Table 1  
Coefficients  $A$  and  $B$  in Eq. (48) for Fig. 6

$Br^*$	$A$	$B$
$10^{-2}$	7.47	6.17
$5 \times 10^{-2}$	7.35	8.21
$10^{-1}$	7.32	9.37
$5 \times 10^{-1}$	7.34	10.78
1	7.37	10.86

Table 2  
Coefficients  $A$  and  $B$  in Eq. (48) for Fig. 7

$Mr^*$	$A$	$B$
1	7.47	6.17
5	7.31	9.46
10	7.19	13.55
15	7.10	17.60

parameters are linearly related to  $Bs^*$  respectively. The correlations are expressed as follow:

$$MBs + MBt = ABs^* + B, \quad (48)$$

where  $A$  and  $B$  denote the slop and interceptor of the lines respectively. The coefficients  $A$  and  $B$  in the above equation are summarized in Table 1 for Fig. 6 and Table 2 for Fig. 7. Using these relations the critical time to mark the onset of convective motion can be found with given conditions.

## 8. Conclusions

In the present study, the onset of solutal Marangoni convection for absorption process accompanied by heat transfer has been investigated under propagation theory.

The following conclusions are drawn from the present study.

1. Based on the scaling analysis, it is found that the sum of the re-modified Marangoni numbers  $MBs$  and  $MBt$  have a linear relation to the modified solutal Biot number  $Bs^*$ . Using these relations, the critical time  $\tau_c$  to mark the onset of Marangoni convection can be predicted with given conditions.
2. There is a critical  $Bs$  causing the most unstable state of fluid layer. This means that the transport condition of the upper gas phase affects Marangoni instability at the surface.
3. The effect of  $Sc$  on the criteria of solutal Marangoni instability is not significant for a larger value than 500.

It is expected that the results from the present study provide a guideline to enhance heat and mass transfer

performance during the absorption process. It can be achieved by performing the numerical simulation of Marangoni convection for the whole time domain using the results of the present study, i.e., the velocity and concentration profiles at the critical onset time as the initial condition of numerical analysis and conducting the experiments on the visualization of Marangoni convection.

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